Temporal Analysis of Synchronizing Pendulums Based On Varying String Length

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Abstract

Christiaan Huygens, in the 1600s, discovered the innate synchronization of coupled pendulums. This paper furthers Huygens's unanswered ambitions by considering the effects of the speed of synchronization based on changes in string length in a coupled pendulum system, designed on a moving platform. Two simple pendulums were connected through the medium of a wooden board which was then placed on cylindrical cans. It's found that string length and synchronization time seemed to display an inverse relationship based on trends of raw data. The findings presented show that synchronization can be optimized which is useful in various fields of study like the medical field where many diseases are caused by the synchronization of neurons. Explanations for other behaviors such as brief stops in motion and anti-phase versus in-phase synchronization are explained using laws of Classical Mechanics and are modeled with polynomial regressions. And the general effect of synchronization arises from the medium between the pendulums and the various dampenings of the system. Finally, the equations of motion and energy are modeled with Lagrangian physics and Mathematica software. Possible extensions, like creating a model similar to the Kuramoto Model, and other applications of the problem are discussed.

***K*eywords** Synchronizing Pendulums · Phase Motion · Euler-Lagrange · Kuramoto Model · Regression

1 Introduction

Synchrony is everywhere in our world. A notable example includes how the circadian rhythm in the body is a group of cells that act as a chronometer to keep the human body in synchrony with the world. There is also the less known synchrony of social phenomena, where one might synchronize the motion of their hands or feet with someone when talking to them in a conversation. In more practical spaces, the vibrations from synchronizing pendulums can be harvested for natural energy [1]. Synchrony in the natural world is not a new topic. But is there a way to optimize this synchronization? That's what this paper aims to find out.

Christiaan Huygens, in the 17th century, discovered that two pendulums can synchronize their motion by setting up the two pendulum apparatuses connected by a wooden beam and releasing the pendulum bobs at different times. The pendulums moved with the exact same speed in opposite directions, mirroring their motion along the y-axis. However, the pendulums need some sort of medium connecting them, otherwise, there is no method of transferring energy which is the method by which they synchronize. That method is also known as the escapement mechanism [2]. If the pendulums synchronized without a medium, then it's either after an elongated period or pure coincidence.

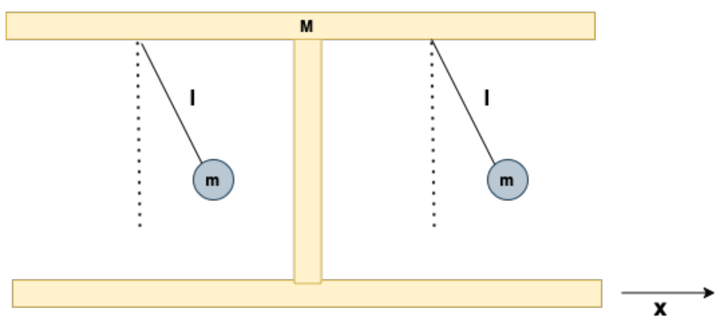

Figure 1: The schematic above shows what Huygens's pendulums would have looked like when he experimented, where M is the total mass of the system, l is the string length, and m is the mass of the bobs [3].

Above in Figure 1, we can see what Huygens's pendulums would have looked like. But unlike the simplified diagram, they were highly mechanical. Unlike the "classical" pendulums we consider in modern times consisting of a string and a bob. The diagram is representative of the rest of his setup. His work involved a wooden beam and the pendulums hanging down from them, but this paper utilized a different approach. The pendulums are built into wooden towers and those towers are placed on a similar wooden board placed on cylindrical cans. This poses no problem as the interest is in the trend in average synchronization times and not the actual numbers from the data (which can change based on the setup utilized in the experiment). With the board below the pendulums, there is more mobility which means that the system will synchronize in a faster time. This is useful for data collection in this experiment since many trials wanted to be performed in little time.

At first, Huygens thought it was air currents causing the pendulums to synchronize [4]. Testing that theory, he found that it didn't work and he died too soon to explore the phenomena any longer. However many questions remain unanswered more than 350 years after Huygens's discovery. What, for example, are the requirements for self-synchronization? Or, what even causes synchronization and how can it be applied? This paper delves deeper into the topic of synchrony to answer Hyugens's ambitions. The pendulums in this paper were arranged in close proximity, and the string length of both pendulums was changed such that both had approximately the same period at all times and the average synchronization times were found, eventually generalizing into a relationship.

In previous research it was shown that there is a transition period for damping coefficient ρ in [0.06, 0.07] there is quasi-periodic motion for in-phase motion [5]. Simply meaning, there is some relation between the material of the medium used and the average synchronization time. We define **in-phase motion** as the pendulums performing the same motions (same period, speed, amplitude, etc.) in the same directions. It follows that **anti-phase motion** are the pendulums performing the same motions but in opposite directions. Huygens's results indicate he used a wood that fell outside the range stated above since he achieved only anti-phase synchronization after hundreds of swings with his pendulums [2]. So in this experiment, wood that has a high enough density to fall within outside that range was utilized. For experimental purposes, Pantaleone created a platform with two metronomes with a phase difference close to 0 [6]. And increasing the damping of the moving platform reported anti-phase motion which is supported by the Kuramoto model.

2 Materials

Below are the materials obtained and utilized for this experiment. Twenty 0.31 cm ($\frac{1}{8}$ in) x 0.31 cm ($\frac{1}{8}$ in) x 91 cm (36 in), ten 0.31 cm ($\frac{1}{8}$ in) x 0.15 cm ($\frac{1}{4}$ in) x 91 cm (36 in) balsa wood sticks (Pitsco Education), 2 Iron Pendulum Bobs (Eisco), 1 Roll of Kite String (HappyToy), 1 Gorilla Glue (Gorilla), 1 Air Dried Poplar Board (Woodcraft, Truevalue), 2 Wooden Shims (Nelson Wood Shims), 4 Mini Metal Staple Hooks (Gardner Bender), 3 Empty Soda Cans (Seagram). Other household items include a hammer, masking tape, pencils, and sandpaper were used during the building. For Data Collection, all that was utilized was a stopwatch. And for Analysis, Mathematica and Wolfram software was utilized.

3 Methods

3.1 Building

The schematic shown in Fig. 2 below has the necessary measurements for the frame of a single pendulum tower. Two frames were made and connected with 4 balsa wood sticks to create the full pendulum tower. This process was duplicated to create another pendulum tower. Fig. 2 shows an 11.43 cm (4.5 in) x 45.72 cm (18 in) rectangle that has rigid cross supports. The diagonal supports of the two separate towers were pointed in opposite directions as shown above. This was to make sure the structure was sound. The measurements of the wider sticks are 0.31 cm ($\frac{1}{8}$ in) x 0.15 cm ($\frac{1}{4}$ in), with any length needed. Having thicker sides lets the structure stand sturdy without much sway. It also prevents the need to create a triangular-shaped tower which is much harder to operate.

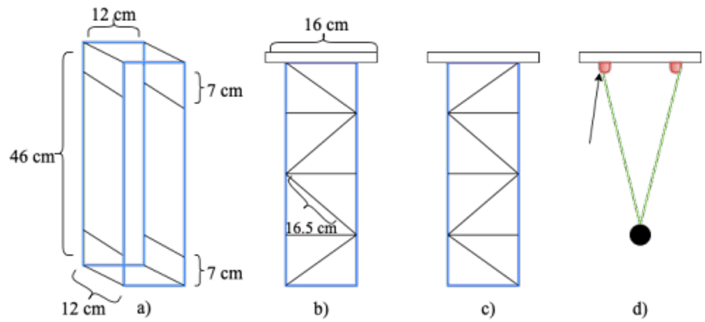

Figure 2: The blue lines designate thick balsa wood sticks, black lines designate the thin balsa wood sticks, green lines designate the string, red lines indicate staple hooks, and the thin rectangles are small wooden shims a) A simplified (doesn't have cross supports) cartoon 3D depiction of the dimensions of the tower (with measurements), including what the sides look like b) The front side of the tower which contains the wooden bar c) The back frame of the tower, the diagonal supports are in the opposite direction of the front frame d) The depiction of how the pendulum bob hangs on the metal hooks (The bob is hooked onto the string).

When gluing the horizontal and diagonal supports, lap joints were avoided as those would reduce the glued surface and bonds. Instead, the edges of the diagonal supports were cut to fit in between the horizontal and vertical supports. When the two rectangular frames were created, they were connected using a singular balsa stick on each side. At the end, the pendulum had the shape of a rectangular prism with dimensions 11.43 cm (4.5 in) x 11.43 cm (4.5 in) x 45.72 cm (18 in). This process was duplicated for a second tower.

Then the mechanism for changing the string length was made. First, a thin wooden bar, made of poplar wood with a width of 3.81 cm (1.5 in) and length of 16.51 cm (6.5 in) was placed at the top of the tower. Two metal staple hooks were hammered into the ends of the wooden bar. Then a long piece of kite string was knotted onto one of the staples and the other end of the string was let loose. That loose end was taped onto the top of the bar, and when adjusting the string was needed, the tape could be undone and the string just needed to be pulled up to shorten the string length. Then the wooden bar was secured onto the top of the tower. Duplicate this process for the second tower. Like in Fig. 3, they were then placed onto a sturdy wooden board. The board rested on three congruent cylindrical cans and two weights were added onto the towers to prevent additional swaying. The cans used were 5.08 cm (2 in) in radius and hollow inside. Using just two cans might damage the cans (from all the weight), thus affecting the results, which is why this experiment utilizes three. Markings for where the cans were be placed so the placement of the cans for each trial is relatively constant. The idea of a moving wooden board was put forward by Kortweg in 1906 [9].

3.2 Procedure

The process began by first pulling one of the pendulums at a certain angle, and then releasing it, letting the pendulums swing on their own until they synchronized as shown in Fig. 3. For clarity, only one pendulum is released and it moves the other pendulum through force. It's known that when the amplitude of the pendulum becomes too large, then the motion of the pendulum becomes uncontrolled. Thus, the standard angle to pull the pendulum was 30 degrees which was marked on the tower itself and was measured with a standard protractor. The poplar board shifted back and forth under the influence of the forces from the heavy pendulum bobs. The board was watched until it came to a complete halt: it was at this point that the timer was stopped. The time on the timer was recorded along with any other observations seen in the pendulum's behavior. This was repeated for 28 trials for each string length. The string lengths ranged from 22.9 cm (9 in) to 36.8 cm (14.5 in) with 1.27 cm (0.5 in) increments which gave 12 string lengths in total. Other behaviors observed include times when one of the pendulums stops and the general nature of the system before and after synchronization. Although, many trials were scrapped for a multitude of reasons. The aluminum cans are very delicate and the setup above the poplar board is quite heavy. The pendulum towers rolled off and the poplar board would have never stopped in some trials. Any trials of that nature were redone. If the poplar board never stopped its motion, then that meant the string lengths of the pendulums were not the same and needed readjustment.

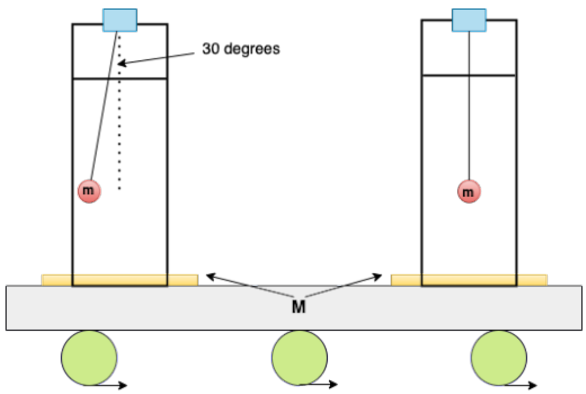

Figure 3: A cartoon depiction of the setup in full (side view, not drawn to scale) and in the starting position. One pendulum is pulled back 30 degrees. The green represents the hollow aluminum cans, red is the iron bobs, yellow is the metal weights to hold down the towers, blue is the wooden plank at the top of each tower which holds the string mechanism, and gray is the poplar board. M and m represent the non-negligible mass of the poplar board + towers and iron pendulum bobs respectively. The arrows at the bottom of the cans represent the friction force.

3.3 Analysis

Once all the trials were collected, for each string length, the trials for each respective string length were all averaged into one number such that there are a total of 12 averages. For the main graph, the average synchronization times (dependent) were on the y-axis while the string length (independent) of the pendulum at a given time was on the x-axis. The ranges of the average synchronization times for each string length were also found after all the trials were recorded. This was to see the standard deviation of the synchronization times which is caused by human error or the other limitations listed later in the paper. Other general observations regarding the nature of synchronization were modeled with Lagrangian Physics. Once the Euler-Lagrange equation was found, Mathematica software was used to analyze the equation of motions. The code is listed in the Discussion.

4 Results

4.1 Qualitative Results

While the pendulums were synchronized, they stayed synchronized for the rest of the motion until the platform ceased movement. The amplitude of the pendulums would gradually decrease while they were moving in their synchronized motion until both pendulums came to a full stop. Something of note is that as with Huygens pendulums, the system proceeded to swing only in anti-phase.

Recalling our definition, this is when the pendulums would move in synchrony with the exact same amplitude and period, but just in opposite directions. The pendulums would never switch between anti-phase or in-phase but just stay anti-phase which was peculiar. When the pendulums were fully synchronized, the platform on which they rested also ceased movement. The procedure for each trial included starting the pendulum with the same amplitude (30 degrees from the vertical), pulling only one of the pendulums out, and releasing it. The pendulum that was first released seemed to stop for a brief moment and then transferred all its former energy to the other pendulum which caused this other pendulum to swing chaotically at full speed. Slowly, the first pendulum would regain its energy and after some time the second pendulum would also experience a brief stop. It was shortly after both pendulums experienced brief stops that the pendulums would synchronize.

4.2 Quantitative Results

22.9 cm (A)	24.1 cm (B)	25.4 cm (C)	26.7 cm (D)	27.9 cm (E)	29.2 cm (F)
37.14	35.16	56.46	47.84	51.81	45.23
40.77	42.76	47.17	35.71	54.37	37.41
48.37	48.34	50.74	37.24	43.33	37.05
50.45	51.25	48.03	40.34	49.91	33.74
59.57	55.47	53.36	51.15	43.83	31.57
53.36	53.26	40.55	49.68	58.78	32.59
59.12	49.09	46.2	51.67	50.22	40.45
51.82	47.83	62.51	64.66	51.43	35.51
59.61	54.32	44.2	63.72	52.8	38.71
56.61	55.71	59.57	65.44	46.55	42.58
56.14	55.41	55.3	61.35	49.68	37.17
60.59	51.39	51.7	54.43	59.04	65.17
59.31	61.89	47.26	62	50.64	50.7
69.87	59.63	46.83	51.15	49.07	38.33
65.8	54.31	45.51	49.68	46.07	34.52
51.29	52.36	43.88	57.67	44.02	33.02
57.56	48.46	42.51	64.72	39.03	34.83
53.47	47.74	41.21	55.44	36.9	32.47
52.55	58.39	43.52	61.35	35.41	50.81
54.28	59.57	42.59	54.43	44.02	58.37
60.29	50.81	58.12	62.89	33.16	50.89
61.76	58.37	64.59	66.93	38.09	46.69
57.91	59.13	61.38	61.08	48.76	37.32
56.82	49.05	56.31	55.97	44.35	35.64
49.86	48.62	49.52	44.03	41.12	38.94
59.43	52.37	52.05	54.77	32.89	39.13
55.66	49.69	44.33	60.14	34.56	42.56
53.21	52.35	49.63	51.91	59.87	44.91
55.45071429	52.24035714	50.17964286	54.90678571	46.06107143	41.26409091

Figure 4: Above are synchronization times (recorded in seconds) each under the column with the associated string length in cm. The measurements (28 trials for each string length) are not necessarily in order of when they were measured. The pink bar represents the average synchronization time for each string length. In general, the relationship seen is decreasing for this first interval of string lengths.

30.5 cm (G)	31.8 cm (H)	33 cm (I)	34.3 cm (J)	35.6 cm (K)	36.8 cm (L)
24.63	29.4	25.38	26.78	34.42	16.45
22.77	33.09	19.33	31.16	33.2	14.36
23.01	30.57	24.72	34.42	24.63	15.21
25.17	31.51	19.52	18.79	28.21	22.09
27.04	31.42	24.67	29.53	37.48	17.85
26.35	27.9	23.52	24.31	27.37	25.12
30.91	20.72	19.01	37.18	30.66	18.53
20.7	21.63	21.72	38.06	31.93	18.04
28.79	23.65	18.8	34.68	26.79	17.26
19.22	30.87	20.58	34.21	31.8	18.5
37.69	32.06	20.13	20.65	27.21	18.33
31.64	34.68	21.42	24.16	31.63	26.82
32.55	31.19	24.05	23.51	14.21	17.99
37.21	32.67	26.42	23.55	14.42	18.37
33.33	30.49	28.72	26.17	19.29	14.7
33.55	30.2	24.6	26.8	14.79	24.21
33.02	38.42	27.77	28.78	14.52	17.12
36.44	32.34	27.46	18.53	15.88	20.56
33.52	31.71	28.07	17.22	25.42	20.37
35.72	27.49	29.15	16.76	24.16	19.77
36.67	24.97	35.16	18.1	17.56	20.89
31.74	35.62	26.54	23.4	19.76	26.97
25.89	25.47	23.45	23.56	20.48	21.34
27.43	23.43	32.43	24.07	18.32	19.08
36.42	29.68	21.94	25.09	17.04	22.78
27.54	34.78	22.87	21.54	21.76	14.98
25.78	22.65	24.06	18.32	16.4	17.29
36.51	23.45	24.39	24.57	14.52	16.54
30.04428571	29.35928571	24.49571429	25.49642857	23.35214286	19.34

Figure 5: This is a continuation of the data from before; and is for string lengths 30.4 cm (12 in) to 36.8 cm (14.5 in). The measurements (28 trials for each string length) are not necessarily in order of when they were measured. The times are measured in seconds, and the pink is the average of all the trials in that column. In general, the relationship seen is decreasing for this interval of string lengths.

The main variable that we were looking at was the effect of string length on the time of synchronization. To recall, each of the string lengths trials were averaged and we ended up with 12 numbers. It was found that string length and average synchronization time has an inverse relationship as shown in Tables 1 & 2. As the string length increases, the average synchronization time tends to decrease. The ranges of each string length were also collected and we list them out here according to their letter: A - [32.73], B - [26.73], C - [24.04], D - [31.22], E - [26.98], F - [33.6], G - [18.47], H - [17.7], I - [16.36], J - [21.3], K - [23.27], L - [12.61]. For the longer string lengths, the ranges are higher than that of the shorter string lengths. In the qualitative observations, the phenomena of brief stops were described and it was also collected numerically. The times at which the brief stops would occur were recorded, but these had much fewer trials since they seemed to have less variance than the average synchronization time. They were also only timed for the first half of the string lengths (A - F) since it was meant as a smaller investigation as part of the larger project.

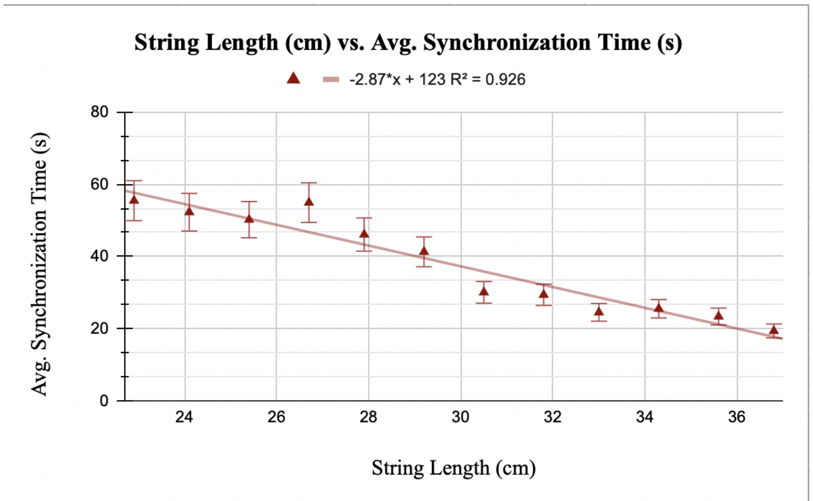

Figure 6: This is the string length (cm) vs. average synchronization times (s) graph taken directly from the data table(s) shown above. The graph includes a polynomial regression as demonstrated by the equation and line of best fit which has a R^2 value of 0.926 which indicates the correlation is very strong. The error bars present on the graph seem to get smaller as the string length increases in length. Overall, the graph represents an inverse polynomial decreasing relationship between the two variables.

22.9 cm (A)	24.1 cm (B)	25.4 cm (C)	26.7 cm (D)	27.9 cm (E)	29.2 cm (F)
11.94	12.34	12.05	10.59	10.68	10.19
10.61	10.45	10.37	10.63	12.01	9.94
10.99	9.85	9.67	11.98	10.32	10.15
10.09	10.67	9.36	9.32	10.91	10.33
9.53	10.09	13.95	8.89	10.31	9.88
10.632	10.68	11.08	10.282	10.846	10.098
20.72	21.84	23.76	22.55	22.66	21.09
20.74	22.45	22.89	21.79	21.29	20.04
19.59	19.09	17.38	23.45	20.83	19.83
18.53	18.31	19.52	19.04	21.91	20.24
18.39	19.67	20.47	18.69	21.66	22.3
19.594	20.272	20.804	21.104	21.67	20.7

Figure 7: This is the table of brief stop times (recorded in seconds) for the pendulums. The yellow section is for the first pendulum and the green is for the second pendulum whereas the average of the brief stops is recorded in pink. Only the first 5 string lengths were used and 6 trials each making a total of 72 trials for the first and second pendulums together. It's observed that the average brief stopping times for each pendulum are all very similar to each other, indicating the relationship is shallow.

5 Discussion

We recall Newton's First Law which states that every object will remain at rest or in uniform motion unless acted upon by an external force [9]. By definition, when coupled pendulums synchronize in anti-phase motion, they have the exact same motion and speed but just in exactly opposite directions. This means that whatever driving forces are exerted on the board from the pendulums are exactly canceled out by each other because they are swinging opposite to each other when in anti-phase synchronization. Thus, if all the forces on the board cancel out, it won't continue to move once it stops. Hence, the pendulums stay in anti-phase motion because there is no external force on the system. That does lead to the thought of why the pendulums are only performing anti-phase motion and not in-phase or perhaps oscillating between the two. It doesn't oscillate between the two states of motion because of the reasoning above. To elucidate, the particular type of wood purchased for this experiment was air-dried poplar wood with a certain damping coefficient. Previous research has pointed out that if the wood has a certain damping coefficient in the interval, then it would perform anti-phase motion which the poplar wood seems to fall under according to our experiments [5]. This indicates that the wood that Huygens used and the wood in this experiment fell under that interval. Also note the fact that during the synchronization process, the pendulum that is released first in the setup stops its motion for a brief moment and continues swinging. At the same time, the pendulum that was not released begins to move at full energy. This is due to the complete energy transfer from the first pendulum to the second pendulum through the media of the poplar board. After the energy transfer, the second pendulum soon transfers its energy back to the first pendulum. It's a curious result that only after both pendulums have transferred energy, not long after, the synchronization of the two pendulums occurs. It would be interesting to see how this would change with more pendulums: would it have to have n (natural number) brief stops for a system of n pendulums? We notice that if the pendulums were simply on the floor, then the brief stops would not happen or be spaced quite far apart as opposed to the data above.

The quantitative results also show that the string length and the average synchronization times are in an inverse linear relationship which is a curious result. An intuitive explanation follows from the basic fact: the period of a pendulum becomes shorter with a shorter string length and longer with a longer string length as inferred from the equation $T = 2\pi\sqrt{\frac{l}{g}}$, where l is string length and g is the gravitational constant [10]. With a shorter period, this means that the pendulum would oscillate more chaotically as opposed to a longer period where the oscillations are more controlled. Thus, if the pendulums's motions are slower and more controlled, it'll be easier to find a point at which their equations of motions intersect than if the motions are fast and chaotic. It's reasoned that the density of the media that the pendulums are attached to also matters. An intuitive explanation is that when the board, as in the setup of this paper, is heavier that means there is more weight for the force of the pendulums to pull. The pendulum's period would be slower and more controlled because there is a force stopping it from swinging freely. Thus, it would synchronize in less time. One could add small weights (ex. hex nuts) to the poplar board and increment the weight to properly examine. Speaking of the inverse relationship, the line of best fit for the graph of the data shows an exponential relationship demonstrating a shallow curve. This might entail the relationship being a shallow curve for all string lengths. Or rather, if we are given a larger set of string lengths, then the relationship may be a steep curve evening into a more shallow one. This indicates that the model might be a good fit for quasi-periodic motion, but to test that requires very large pendulums of great cost. In fact, we notice that the polynomial and linear approximations give the same R^2 value, but this is due to the fact we modeled a smaller subset of string lengths. The polynomial representation is picked because it would be more accurate for quasi-periodic motion. More statistically, it's noticed that the ranges of the average synchronization times also get more constricted as they go on in the list. It's also noticed that the error decreases as the string length increases, which demonstrates the same inverse relationship that the main finding has. As the string length gets longer, the motion is more controlled so the synchronization times would tend to happen quicker and in a smaller range.

For reference, the motion of the pendulums can be modeled with Lagrangian mechanics. This is more approachable in this case as opposed to Newtonian mechanics because we don't have to consider the forces of the constraints. We have to consider the Kinetic and Potential energy for the equation L = T - V. We will have to use $T = \frac{1}{2}Mx^2$ where M is the mass of the poplar board and x is the position vector following, $x =< r + cos\theta, sin\theta >$ When combining all the factors and taking into account all the different parts in the tower, we obtain Equation 1. The variables are as follows: m is the mass of the pendulum bob, M is the mass of the system, θ_1 and θ_2 are the angle displacements of the pendulums, l is the string length, and k is the "spring constant."

$$L = \frac{1}{2}(M + 2m)(\dot{x}^2) + m\dot{x}l(\theta_1 cos(\theta_1)$$

$$+\theta_2 cos(\theta_2)) + \ ml^2(\theta_1^2 + \theta_2^2)$$

$$+mgl(cos(\theta_1) + cos(\theta_2)) - \frac{1}{2}kx^2$$

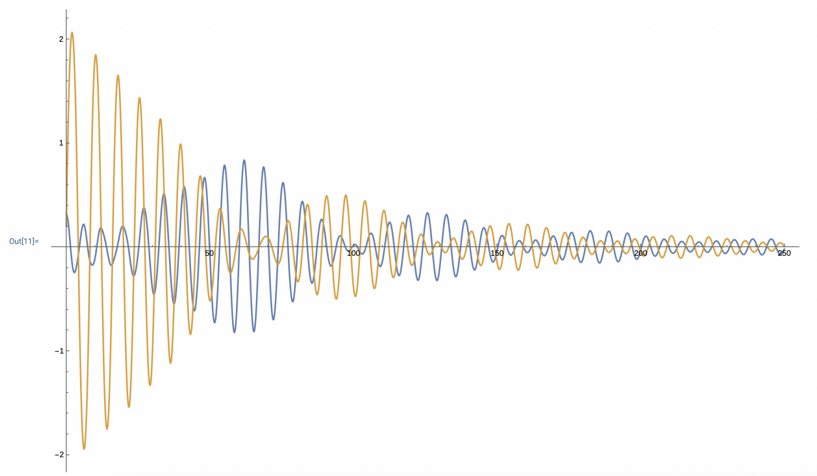

Figure 8: Above is a graph of the Euler Lagrange Equations modeled using the Mathematica software. Damping is accounted for through a constant in the form of bx in the Mathematica code which can be found in the Appendix. The Euler-Lagrange equations are the solutions to the general equation of motion of the system given above. The damping constant makes it so that the equations approach each other, as shown in the graph. As they approach each other, it looks as if they are going into anti-phase motion, which explains why our pendulums appeared to only be going in anti-phase as opposed to in-phase.

The code to generate the above graph is given below.

```
sol = NDSolve[{30 x''[t] + y''[t] Cos[y[t]] + z''[t] Cos[z[t]] -
     y'[t]^2 Sin[y[t]] - z'[t]^2 Sin[z[t]] + 30 x[t] + 2 x'[t] == 0,
    y''[t] + x''[t] Cos[y[t]] + Sin[y[t]] - x'[t] y'[t] Sin[y[t]] +
     0.02 y'[t] == 0,
    z''[t] + x''[t] Cos[z[t]] + Sin[z[t]] - x'[t] z'[t] Sin[z[t]] +
     0.02 z'[t] == 0, x[0] == 0, x'[0] == 0, y[0] == Pi / 10,
    y'[0] == 0, z[0.5] == 1, z'[2] == 0}, {x, y, z}, {t, 0, 1000}]
Plot[{Evaluate[y[t] /. sol], Evaluate[z[t] /. sol]}, {t, 0, 250},
  PlotRange → All]
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The equation above doubly serves as the motivation for testing on the string length in the first place, as the equation of motion includes the string length, l. Overall, much testing can be done on the equation of motion. Intuitively, the phase difference between the pendulums should drift in a periodic way, in an ideal situation, with the absence of any dissipative effects. That is what you get when you plug in the Euler - Lagrange equations into Mathematica. They seem to demonstrate periodic behavior, but there is no synchronization that way. Recall that damping is a term that is demonstrated as proportional to velocity.

Such a term can be added to either of the Lagrange equations, and it will change the graphs such that they synchronize as shown above in the graph. In the graph, the equations tend towards each other and the curves get increasingly similar. We also notice that the equations are intertwining in an anti-phase pattern. Intuitively, this makes sense since when adding a damping term to the equation, the scope of the y values is smaller, thus an intersection between the graphs would happen. It's known that wetting the surface that the pendulums are rolling on also changes the synchronization of the pendulums since damping is increased [11]. It's a known fact that adding more damping can cause synchronization, but looking at the equations of motion is an obscure way to demonstrate this fact.

Recent applications of synchronizing pendulums have risen in the medical and engineering professions. In a recent study, it was shown that neurons synchronize their beats like the pendulums in Huygens experiments. Neurons synchronizing their beats are seen in quasi-rhythmic activities where brain waves are generated. For example, in the hippocampus (responsible for memory), brain waves undergo theta synchronization which ensures the encoding of episodic memories [12]. Certain diseases mess with the synchronization of these neurons affecting the memory [12]. For example, it is speculated that epilepsy occurs because of the synchronization of neurons that take place in the brain. As shown above, since longer pendulums synchronize faster, how would the prolongation of an electrical impulse from a neuron nucleus (or soma) to the axon affect the neuron(s) sychronization? Of course, the connection between these two fields would need further study, but it's an interesting idea to think about. The results in this experiment show that synchronization can be optimized which is useful for certain inventions that rely on modifying certain properties of synchronization.

This problem has many extensions since pendulums are so adaptable and have a multitude of features. For example, how is synchronization changed by the pendulum bob weight or by the mobility of the medium? Although like with every question, there may come some limitations. For example, with the pendulum bob weight, the towers would need to be made of a thicker wood for it to hold a large weight of metal. For adjusting the mobility of the board, the board would need to be extremely long to get a good sample size. Many trials would have to be done as well since the cans were hard to work with even just three of them. Pendulums serve as better synchronizers for experiments than metronomes because pendulums have many factors and are not self-driven. Expanding on the explanation above, an interesting expansion would be seeing how synchronization is affected by the distance between two pendulums. Does distance have a factor in synchronization? One question that is best left untouched would be trying to adjust the string length so that the pendulum string lengths differ at all times. For example, you could see how a difference of 1 in, 2 in, 3 in, etc. between the string lengths of the pendulums changes the synchronization time of the pendulums. The problem is that the pendulums wouldn't be able to synchronize with different string lengths because the periods would be different. For the two pendulums to synchronize, they need to move in the exact same motion with the exact same speed. They can only have the same speed if they have the same period. Thus, the poplar board would never cease movement. Despite that, there are many avenues for exploration with synchronizing pendulums. Particularly, if the goal is to find a model for simple pendulums like the Kuramoto model for chaotic oscillators, then experimenting with a different amount of pendulum is vital. Generalizing the investigations to N pendulums is crucial. Overall, finding more factors can assist in creating a mathematical model for oscillators with a non-intrinsic frequency.

6 Limitations

The towers were built with thick balsa wood and many connecting pieces to reduce the amount of internal friction and unnecessary swaying of the towers. Since the towers were man-made, they were not completely free of human error in the building process which may have caused small shifts in the board. This could have either hastened or slowed the synchronization times of the trials. In some trials, the board would never fully stop and keep on moving even after the pendulums synchronized, but those trials were disregarded so they never posed a problem. The surface on which the pendulums were placed was smooth to ensure the negligibility of friction. Since the timing was also done by a human, there is human error, but the timing and the releasing of the pendulum were done by the same person to reduce uncertainty. One of the bigger sources of error would deal with the cans placed under the wooden board that held the pendulums. The cans were made of thin aluminum and the rolling could have easily created a dent or two in them while experimenting. The dents would have caused the board to roll less efficiently and not accurately, possibly affecting the synchronization time. This was somewhat mitigated by checking the cans and a trial was scrapped if the cans were deemed dented. In this experiment, the trials are not based on human surveys or human subjects so there is no apparent bias. Overall, much of the possible error was mitigated to ensure the accuracy of the experiment.

7 Conclusion

It was found that the string length of a coupled pendulum system has an inverse linear relationship with the average synchronization time of pendulums. Other results like the fact damping indeed causes synchronization are confirmed through analysis in Mathematica. Other smaller results with the nature of the synchronization of the system are also discussed and all are analyzed using laws of classical mechanics. As the string length of the pendulums gets longer, the time for the pendulums to synchronize is shorter. A pendulum is an oscillator, meaning it has a specific period, but its non-intrinsic frequency makes pendulums an interesting subject. Since they don't have their own frequency, it makes it easier to apply the results of this paper and similar investigations to other real-world phenomena that have synchronization. For example, in the medical field, synchronization of pendulums can help understand how diseases work. In fact, many of them, like epilepsy, are based on the synchronization of neurons which triggers intense reactions. Pendulums also have more properties which make it easier to change the average synchronization time and add more to the model that this paper works towards. In general, the idea of properties affecting synchronization means that synchronization can be optimized, which is the essence and main idea of the research presented in this paper. There is so much that goes on even with just the string length of a pendulum and synchronization in general, as this paper presents. The research presented is pure physics, thus it serves as a gateway for even more exploration regarding this topic and highlights the need for further research in this area.

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